Medians and altitudes of a triangle (pp 279-281)

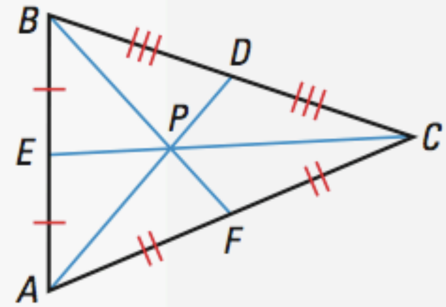
THEOREM

THEOREM 5.7 ***Concurrency of Medians of a Triangle***

The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

If P is the centroid of $\triangle ABC$, then

$AP = \frac{2}{3}AD$, $BP = \frac{2}{3}BF$, and $CP = \frac{2}{3}CE$.

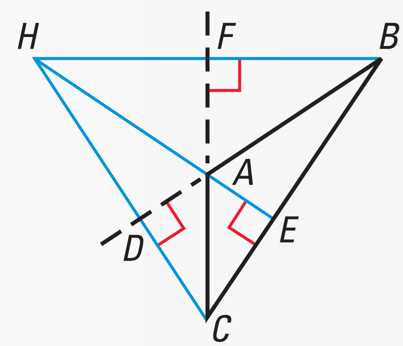


THEOREM

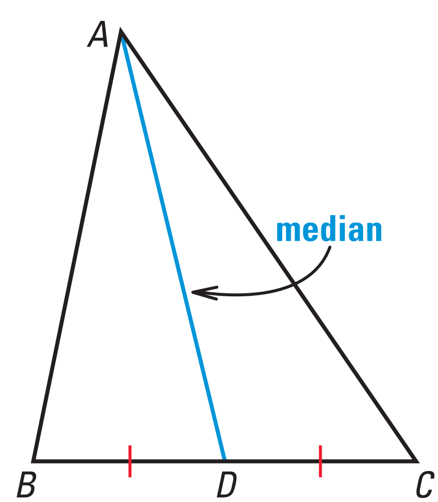
THEOREM 5.8 ***Concurrency of Altitudes of a Triangle***

The lines containing the altitudes of a triangle are concurrent.

If $\overline{AE}$, $\overline{BF}$, and $\overline{CD}$ are the altitudes of $\triangle ABC$, then the lines $\overleftrightarrow{AE}$, $\overleftrightarrow{BF}$, and $\overleftrightarrow{CD}$ intersect at some point H.



A **median of a triangle** is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side. For instance, in $\triangle ABC$ shown at the right, D is the midpoint of side $\overline{BC}$. So, $\overline{AD}$ is a median of the triangle.



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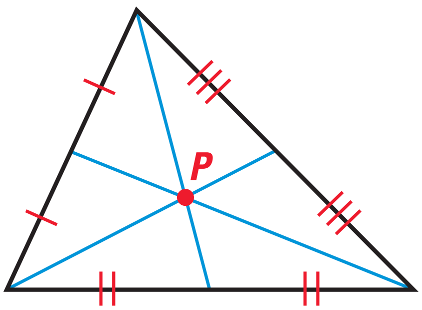
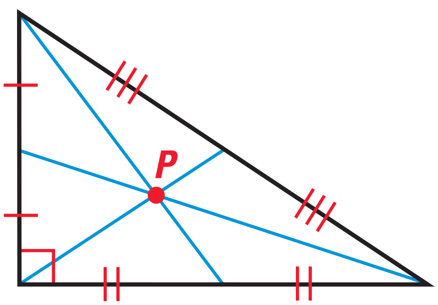
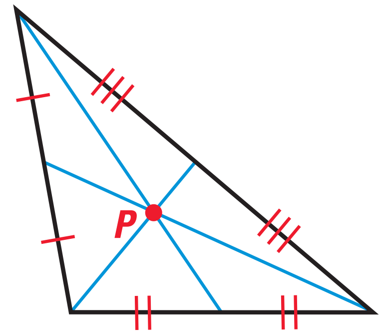
The three medians of a triangle are concurrent. The point of concurrency is called the **centroid of the triangle**. The centroid, labeled *P* in the diagrams below, is always inside the triangle.



acute triangle

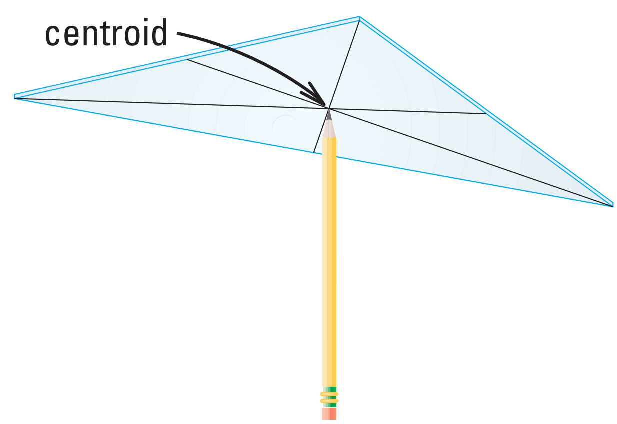


right triangle



obtuse triangle

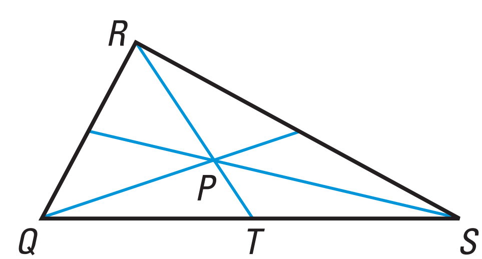
A triangular model of uniform thickness and density will balance at the centroid of the triangle. For instance, in the diagram shown at the right, the triangular model will balance if the tip of a pencil is placed at its centroid.



EXAMPLE 1 *Using the Centroid of a Triangle*

P is the centroid of $\triangle QRS$ shown below and $PT = 5$. Find RT and RP.

SOLUTION



Because P is the centroid, $RP = \frac{2}{3}RT$.

Then $PT = RT - RP = \frac{1}{3}RT$.

Substituting 5 for PT, $5 = \frac{1}{3}RT$, so $RT = 15$.

Then $RP = \frac{2}{3}RT = \frac{2}{3}(15) = 10$.

▶ So, $RP = 10$ and $RT = 15$.

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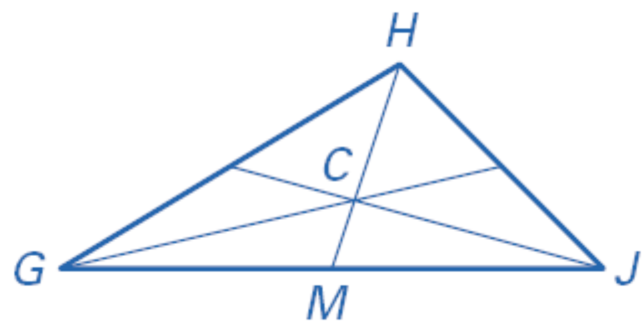
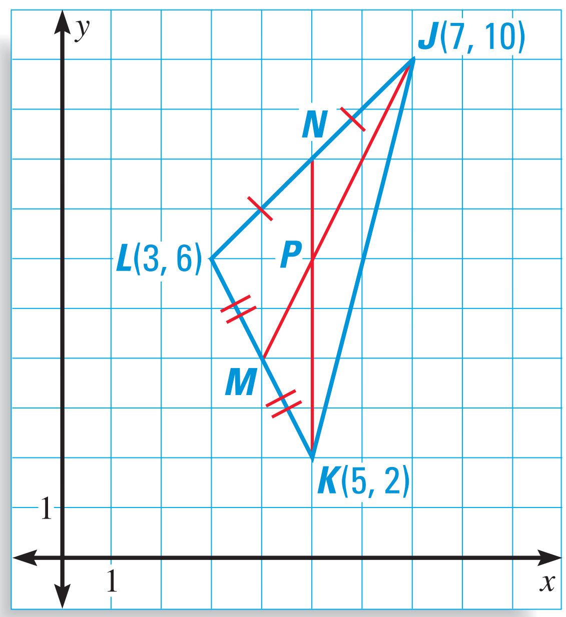
1. C is the centroid of $\triangle GHJ$ and $CM = 8$. Find HM and CH.



EXAMPLE 2 Finding the Centroid of a Triangle

Find the coordinates of the centroid of $\triangle JKL$.



Solution

You know that the centroid is two thirds of the distance from each vertex to the midpoint of the opposite side.

Choose the median $\overline{KN}$. Find the coordinates of N, the midpoint of $\overline{JL}$. The coordinates of N are

$$\left(\frac{3+7}{2}, \frac{6+10}{2}\right) = \left(\frac{10}{2}, \frac{16}{2}\right) = (5, 8).$$

Find the distance from vertex K to midpoint N. The distance from $K(5, 2)$ to $N(5, 8)$ is $8 - 2$, or 6 units.

Determine the coordinates of the centroid, which is $\frac{2}{3} \cdot 6$, or **4** units up from vertex K along the median $\overline{KN}$.

▶ The coordinates of centroid P are $(5, 2 + \mathbf{4})$, or $(5, 6)$.

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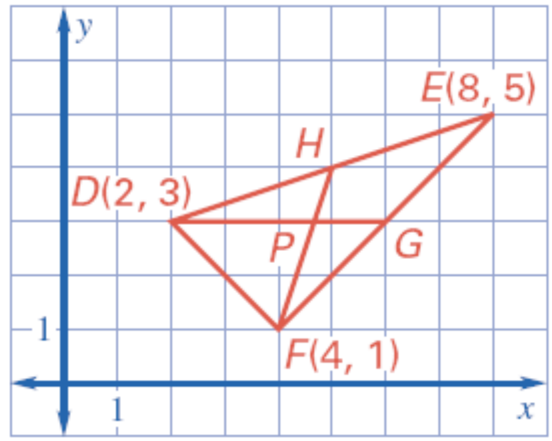
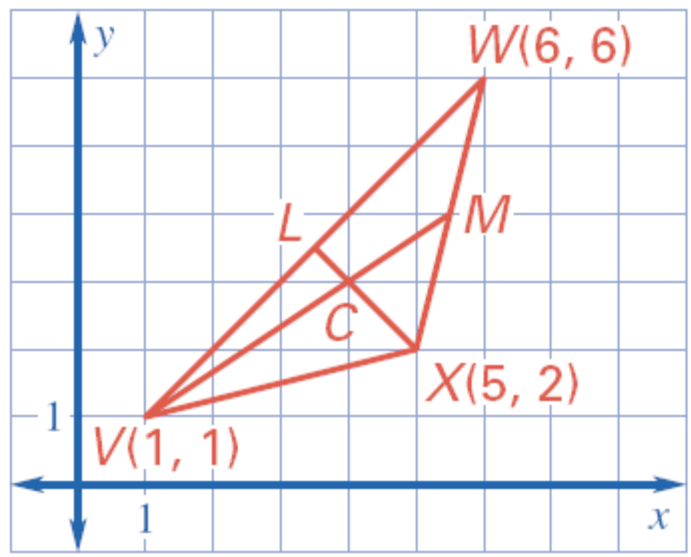
2. Find the coordinates of centroid P of $\triangle DEF$.



Point C is the centroid of $\triangle VWX$ **.**

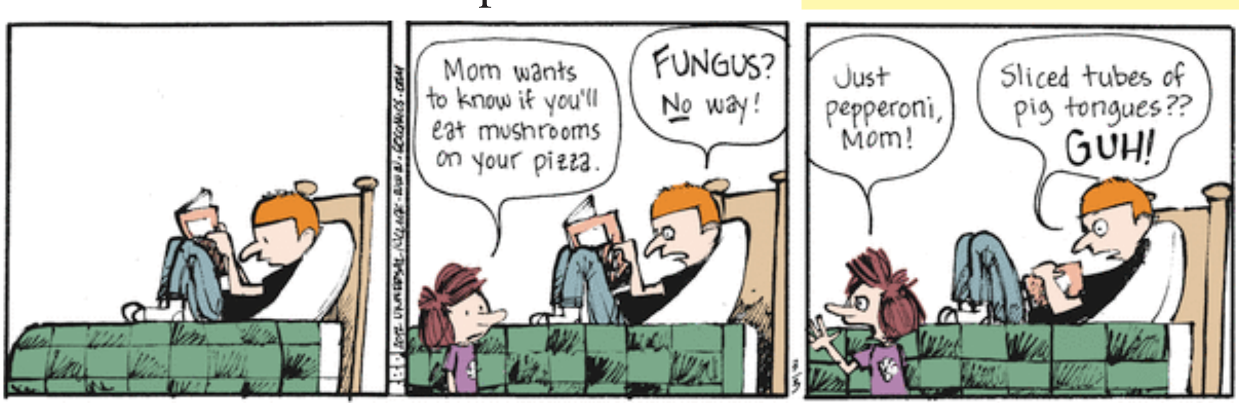
1. Find the coordinates of point C.

2. If $CL = \frac{\sqrt{2}}{2}$, find CX.



An **altitude of a triangle** is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side. An altitude can lie inside, on, or outside the triangle.

Every triangle has three altitudes. The lines containing the altitudes are concurrent and intersect at a point called the **orthocenter of the triangle**.



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Logical Reasoning

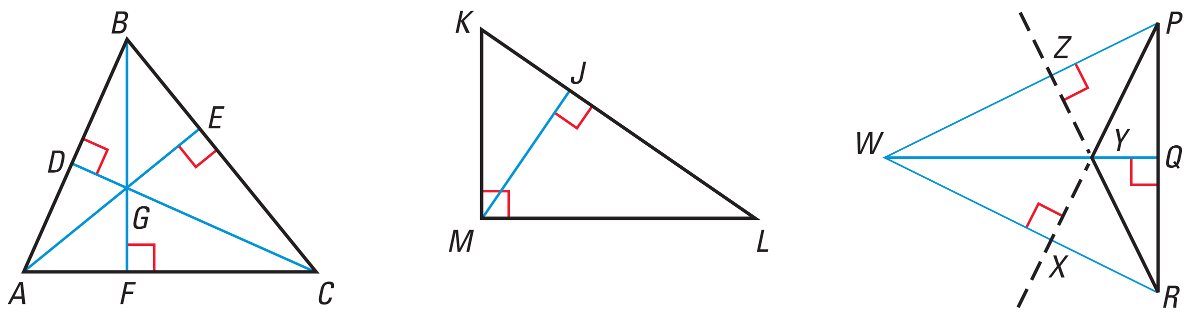
EXAMPLE 3 *Drawing Altitudes and Orthocenters*

Where is the orthocenter located in each type of triangle?

a. Acute triangle **b.** Right triangle **c.** Obtuse triangle

SOLUTION

Draw an example of each type of triangle and locate its orthocenter.



a. $\triangle ABC$ is an acute triangle. The three altitudes intersect at G, a point *inside* the triangle.

b. $\triangle KLM$ is a right triangle. The two legs, $\overline{LM}$ and $\overline{KM}$, are also altitudes. They intersect at the triangle's right angle. This implies that the orthocenter is *on* the triangle at M, the vertex of the right angle of the triangle.

c. $\triangle YPR$ is an obtuse triangle. The three lines that contain the altitudes intersect at W, a point that is *outside* the triangle.

Where is the orthocenter located in $\triangle ABC$?

3. If $m\measuredangle A = m\measuredangle B = m\measuredangle C$?

4. If $m\measuredangle A = m\measuredangle B = 45°$

5. If $m\measuredangle A = 110°$

6. What line segments intersect to form a centroid?

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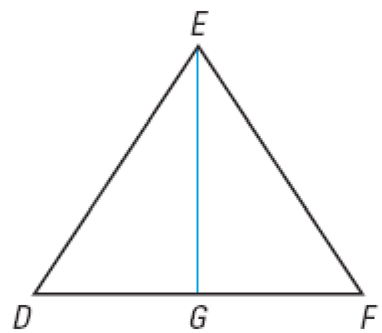
Use the diagram and the information to decide in each case whether $\overline{EG}$ is the given segment. More than one answer may be possible. Choose all that must be true.



7. _____ $\overline{DG} \cong \overline{FG}$

A. Perpendicular bisector
B. Angle bisector
C. Median
D. Altitude

8. _____ $\overline{EG} \perp \overline{DF}$

A. Perpendicular bisector
B. Angle bisector
C. Median
D. Altitude

9. _____ $\measuredangle DEG \cong \measuredangle FEG$

A. Perpendicular bisector
B. Angle bisector
C. Median
D. Altitude

10. _____ $\overline{EG} \perp \overline{DF}$ & $\overline{DG} \cong \overline{FG}$

A. Perpendicular bisector
B. Angle bisector
C. Median
D. Altitude

11. _____ $\triangle DGE \cong \triangle FGE$

A. Perpendicular bisector
B. Angle bisector
C. Median
D. Altitude